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Magnetic Coupling To A Loop Antenna In Proximity Of Shorted-large Loops

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1 Introduction

In operations that involve explosives, magnetic fields (B) are a concern because of induced voltages on detonator cables. The fields could be generated by a nearby lightning strike, or even within a "Faraday cage" struck by lightning [1.1]. The wavelength of the lightning induced fields is much longer than typical detonator cables [1.2]. A hypothetical example consisting of explosives in a drum initiated by the detonator at the top is shown in Figure 1.1. This configuration without an initiation system is normally considered safe. However, given a magnetic field, a cable voltage (V_{loop}) will be created. It can be computed from the following equation that assumes the cable naturally forms a loop of Area_{cable} [1.3].

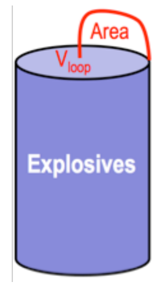


Figure 1.1. Explosives and detonator cable.

$$V(t)_{loop} = -\text{Area}_{cable}\frac{dB}{dt} \qquad (1.1)$$

For safety reasons, calculating this voltage accurately is important. If there are metal structures consisting of shorted-large loops around the explosives, the calculation becomes much more complex. The voltage could be significantly higher or lower than that calculated by eq. 1.1. In Figure 1.2, the hypothetical explosives are mounted on a work stand consisting of metal beams that create two shorted-large loops. The loop formed by the cable and drum may have many non-conduction gaps: e.g., between the detonator bridge-wire and drum, and the open end of the cable to drum or work stand. The loop voltage would divide according to the capacitances of the gaps. For simplicity, in this report the loop voltages in the gaps will be lumped into one voltage in one gap. In the didactic examples, the drum is also removed.

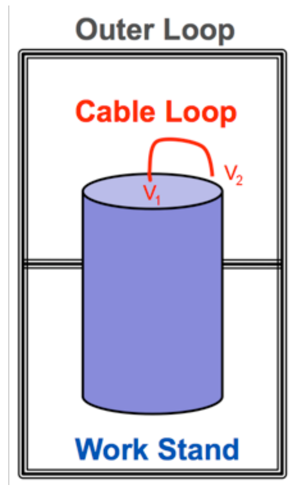


Figure 1.2. Explosives are supported in a work stand.

The goal of this report is to explain how to estimate the cable voltage in a changing magnetic field surrounded by a metal structure. Two models, full and simplified, have been developed and validated by laboratory studies. The full treatment of the coupling problem consists of analytical equations, 3D electromagnetic computational coupling models, and circuit models will be published [4]. This report covers the simplified equations based on flux density calculations in Section 2. These simpler equations produce rough estimates that can establish the justification for the full analysis. We will start with generic examples to illustrate the application of the theory, and practical configurations will be covered at the end of the section. Laboratory studies were performed on canonical loops to validate the analysis, and the results are reported in Section 3. A computer simulation shows in Figure 1.3 the concentration of magnetic flux lines around a shorted-large loop and an open-small loop on the lower left when exposed to a spatially uniform and time-varying magnetic field. An objective of this report is to provide an intuitive understanding of the pattern in the figure and how the voltages are generated.

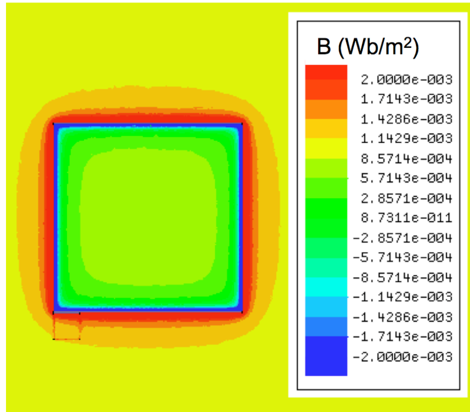


Figure 1.3. Computer simulated magnetic field.

2 Simplified Equations

The two objectives of this section are to provide (1) an intuitive understanding of how large loops interact with smaller loops and (2) a simple formula to estimate the open-small loop voltage. In order to offer simplified equations, we must make three assumptions. (a) We will concentrate on rudimentary square loops, (b) of practical sizes in the range of inches to feet. (c) It is assumed that the inductive impedance of the loops is much greater than the resistance, and it will be eliminated in the calculations. Within the dimensional constraints, the simple formula will produce a loop voltage that should be accurate within a factor of two of the real value. Two different configurations will be compared: (1) separated loops and (2) touching loops, and (3) practical considerations will be discussed.

2.1 Separated Loops

The open-circuit voltage and short-circuit current for a small and large square loop will be compared. (See Figure 2.1.) The small loop has a side length of “Len”, and the large loop is “x” times the small-loop dimension. The gap voltages in the open loops are:

$$V(t)_{small} = -\text{Len}^2 \frac{dB}{dt} \qquad (2.1)$$

$$V(t)_{large} = -\left(\text{x Len}\right)^2 \frac{dB}{dt} = \text{x}^2\, V(t)_{small} \qquad (2.2)$$

Note that the large-loop voltage is increased by the square of the length ratio, “x”.

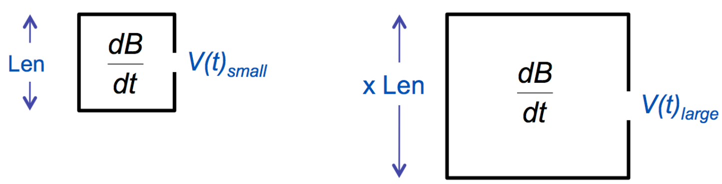


Figure 2.1. The large-loop dimension is “x” times the small-loop length.

The short-circuit current is predominately limited by inductance of the loop, and hence is proportionate to the length of the loop. The closed loop current (i) is determined by the flux (Φ) within the loop divided by the inductance (L). The inductance for practical size loops can be estimated from the free-space inductance. If the small-loop inductance is L, the large-loop inductance is about x L. For the small loop, the flux is equal to the area, Len^2, times the flux density (B). The current in the large loop is "x" times the current in the small loop.

$$i(t)_{small} = \frac{\Phi(t)_{small}}{L} = \frac{Len^2}{L} B(t) \qquad (2.3)$$

$$i(t)_{large} = \frac{\Phi(t)_{large}}{x\,L} = \frac{x^2\,Len^2}{x\,L} B(t) = \frac{x\,Len^2}{L} B(t) = x\,i(t)_{small} \qquad (2.4)$$

For the case where "x" is two, the large-loop voltage is 4 times higher than the small-loop voltage. The current is only twice as high. The voltage and current scales differently from the small to the large loop, and the point becomes important for the touching loops.

$$V(t)_{large} = -\,2^2\,Len^2\,\frac{dB}{dt} = 4\,V(t)_{small} \qquad (2.5)$$

$$i(t)_{large} = -\,\frac{2\,Len^2}{L} B(t) = 2\,i(t)_{small} \qquad (2.6)$$

An argument was made earlier that the inductance of the loop could be estimated by the free-space permeability times the circumference. A commonly used inductance formula for a square loop [2.1] and the free-space estimate are:

$$L_{square} = \mu_0\,\frac{2\,Len}{\pi}\left[\ln\left(\frac{Len}{radius}\right) - 0.774\right] \qquad (2.7)$$

$$L_{sq\text{-}free\text{-}space} \approx \mu_0\,4\,Len \qquad (2.8)$$

For a loop with a wire radius of 1 mm, the wire in free-space inductance and the square loop inductance are plotted in Figure 2.2. The use of free-space wire inductance is a reasonably close approximation for human-size loops and will provide a more intuitive understanding of loop voltages and currents.

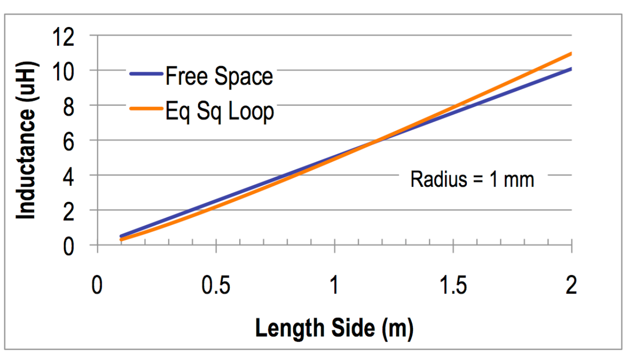


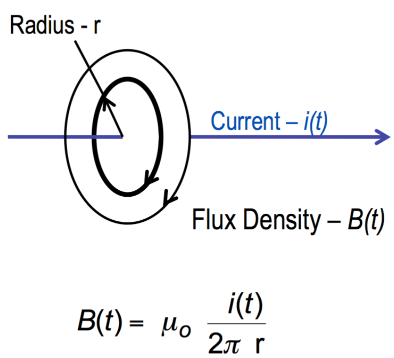
Figure 2.2. For human-size loops, the free-space inductances and square loop inductances are similar.

2.2 Touching loops

To understand the magnetic coupling of touching loops, we must investigate the flux around a wire generated by a current. (See Figure 2.3.) The flux density drops off by the inverse of the radius [2.2]:

$$B(t) = \mu_0 \frac{i(t)}{2\pi\, r} \qquad (2.8)$$

The flux density is plotted in the figure along with the normalized cumulative flux from 1 mm to 5 cm. This concentration of the flux around the shorted-large loop is the mechanism that could significantly increases the detonator cable voltage.



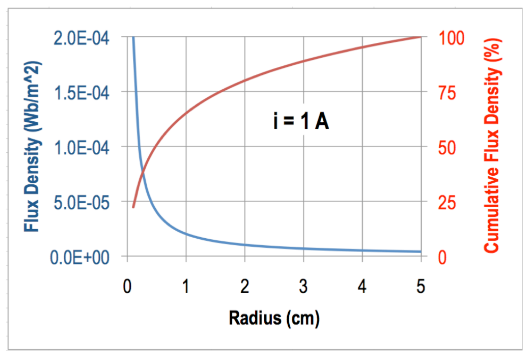


Figure 2.3. Flux density drops quickly away from the wire.

The touching-loop configuration is shown in Figure 2.4. The two loops are in the same x-y plane, and the uniform magnetic field is pointed out in the z-direction. The touching, or shared, wire is in the x-direction. The shorted and open loops have lengths of $Len_{shorted}$ and Len_{open}, respectively, and shorted- or open-loop wire radius, R. The open-loop voltage is determined by the rate of change of the total flux. Because the small loop is touching on the outside of the large loop, the total flux consists of the external field and the magnetic field generated by the short-circuit current in the large loop:

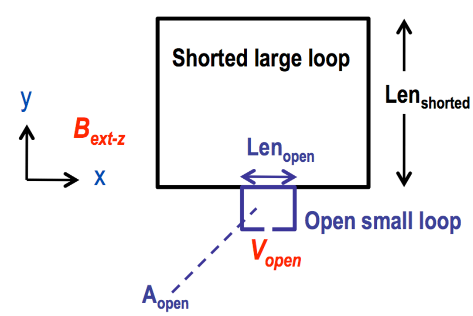


Figure 2.4. Touching-loop configuration.

$$V_{open} = -\frac{d\,\Phi_{total-open}}{dt}\text{ , where} \qquad (2.9)$$

$$\Phi_{total\text{-}open} = \int_{A_{open}} B_{total-open}\, dA = \int_{A_{open}} \left(B_{ext} + B_{shorted}\right) dA \qquad (2.10)$$

The objective is to solve the voltage equation in terms of physical loop dimensions and the magnetic field. Considering the flux equation, two simplifications will be helpful: Only the flux from the shared wire portion of the large loop will be counted, i.e., the contribution from the perpendicular and far parallel wires of the large loop will be ignored. We will again use the wire in free-space inductance approximation. The total flux is approximated by:

$$\Phi_{total\text{-}open} = B_{ext}\ \mathrm{Len}_{open}^2 + \mathrm{Len}_{open} \int_{R}^{\mathrm{Len}_{open}} B_{shorted-y}\ dy, \qquad (2.11)$$

assume $L_{shorted} \approx \mu_o\ 4\ \mathrm{Len}_{shorted}$ and applying [2.3]

$$B_{shorted-y} = \mu_o \frac{i_{shorted}}{2\pi y} = \mu_o \frac{\mathrm{Len}_{shorted}^2}{2\pi y\ L_{shorted}} B_{ext} \approx \frac{\mathrm{Len}_{shorted}}{8\pi y} B_{ext} \qquad (2.12)$$

$$\Phi_{total\text{-}open} \approx B_{ext}\ \mathrm{Len}_{open}^2 + \mathrm{Len}_{open} \int_{R}^{\mathrm{Len}_{open}} B \frac{\mathrm{Len}_{shorted}}{8\pi y} B_{ext}\ dy \qquad (2.13)$$

$$\approx B_{ext}\ \mathrm{Len}_{open}^2 + \frac{\mathrm{Len}_{open}\ \mathrm{Len}_{shorted}}{8\pi} B_{ext} \int_{R}^{\mathrm{Len}_{open}} \frac{1}{y}\ dy \qquad (2.14)$$

$$\approx B_{ext}\ \mathrm{Len}_{open}^2 + \frac{\mathrm{Len}_{open}\ \mathrm{Len}_{shorted}}{8\pi} B_{ext}\ \ln\left(\frac{\mathrm{Len}_{open}}{R}\right) \qquad (2.15)$$

assuming $\mathrm{Len}_{open} >> R$

Substituting the flux equation 2.15 into 2.9, the open-small loop voltage is:

$$V_{open} \approx -\left[\mathrm{Len}_{open}^2 + \frac{\mathrm{Len}_{open}\ \mathrm{Len}_{shorted}}{8\pi}\ \ln\left(\frac{\mathrm{Len}_{open}}{R}\right)\right] \frac{d\,B_{ext}}{dt} \qquad (2.16)$$

The first term accounts for the flux captured by the just the open loop, without the shorted loop. The second term describes the flux generated by the shorted loop that is captured by the open loop. The natural logarithmic function (ln) in eq. 2.16 complicates our understanding and is plotted in Figure 2.5. The open-loop length to wire radius ratio is compressed by the function. For our type of application, the log function will return a number between 1 and 10.

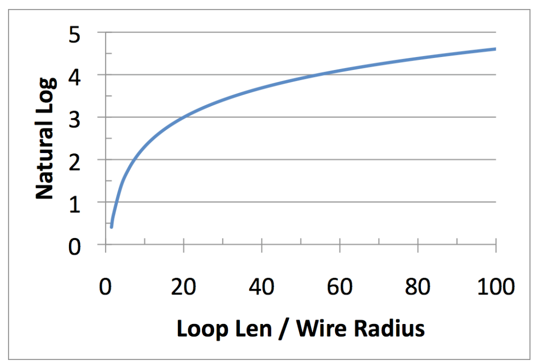


Figure 2.5. The loop size to wire ratio is compressed by the natural logarithmic functions.

There are limitations to the use of the simplified equation, such as the open loop must be smaller than the shorted loop. If the small-open loop is inside of the shorted-large loop, the sign of the second term in eq. 2.16 must be reversed. If the small loop is inside the shorted loop, and is near or touches the corner, the equation will under estimate the voltage because of the higher flux density in the corner. In general, locations outside and adjacent to the shorted-large loop have higher total flux density, and thus produce more voltage, than inside the shorted loop. (See Figure 1.3.)

At this point, some examples should be useful. Based on Sandia National Laboratory analysis of a facility struck by lightning, we will assume a magnetization rate (dB/dt) of 2 kWb/m^2s from an extreme strike. In Figure 2.6, the voltage for various size open loops is plotted against the shorted-large loop size. The wire radius is assumed to be 1 mm. The voltage rises linearly with large-loop size because it is not in the natural log term. Extrapolation of the lines to zero shorted-loop length gives the contribution from just the open loop. For example, the 10 cm length open-loop voltage denoted by the green line would generate 20 V away from the large loop. However, the voltage is increased by a factor of 4, to 80 V, if it shares a wire with a large loop of 1.5 meters on a side. Depending on the detonator, this common-mode voltage might be significant.

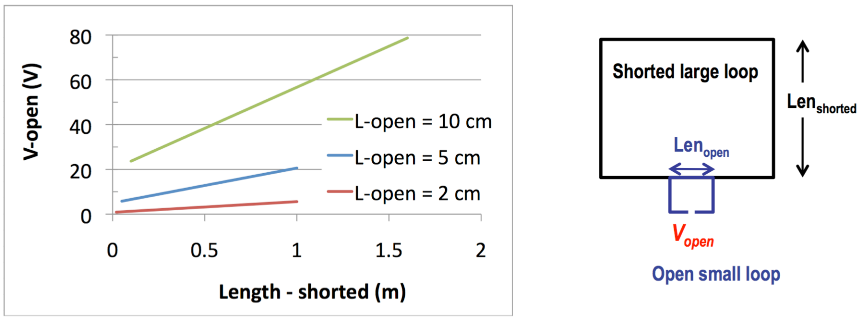


Figure 2.6. The small-loop voltage increases linearly with large-loop size.

A plot of the gap voltage, as a function of the small-loop size, is more complex. (See Figure 2.7.) The lowest, black-dotted, line denotes the small-loop voltage as a function of size without the influence of the shorted-large loop. As the size of the shorted-large loop increases, the contribution from the term with natural log function dominates the output voltage.

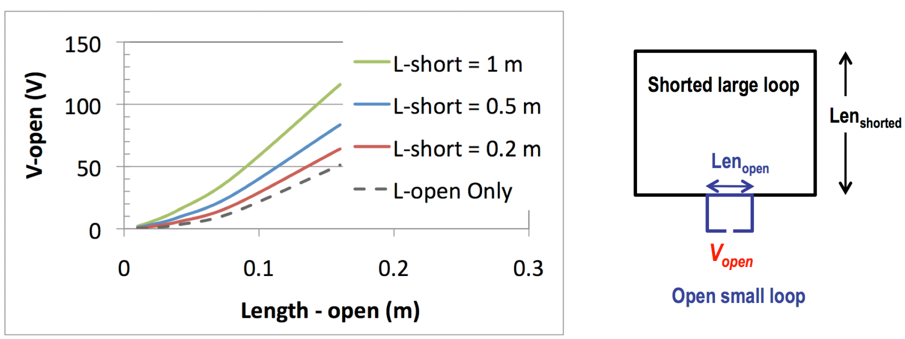


Figure 2.7. The small-loop voltage increase with size is complex.

2.3 Practical Considerations

While the square loops interactions can be described by simple equations, rectangular shaped loops with large "wire" sizes are more common. The rectangular loop voltages can still be estimated from the square loop equations without resorting to more complex and less intuitive equations. Three configurations were examined: rectangular open loop, rectangular shorted loop, and "fat" wires.

The rectangular open-loop configuration is shown in Figure 2.8. The voltage of the small loop can be estimated from the voltage of a square loop of equivalent area multiplied by a geometric correction factor, $k_{rect\text{-}geo}$ defined in eq. 2.17. The voltage contribution from solely the small rectangular loop is the same as the square loop. However the contribution from the shorted-large loop increases because more flux lines will be captured by the wider open loop.

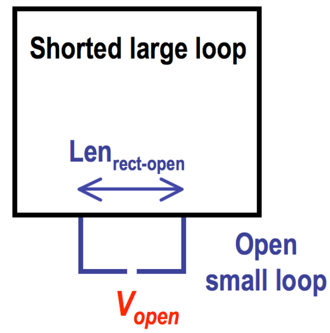


Figure 2.8. Rectangular open-small loop.

$$V(t)_{rect-open} \approx V(t)_{sq-open}\, k_{rect\text{-}geo}\ , \quad \text{where } k_{rect\text{-}geo} = \frac{Len_{rect\text{-}open}}{Len_{sq\text{-}open}} \qquad (2.17)$$

A rectangular shorted-large loop with the same area as a square loop will produce a lower magnetic field. While the induced voltage in the two large loops is the same, the inductance of the rectangular loop is higher because of the longer perimeter. Hence, the short-circuit current is lower, and the magnetic fields will be lower. Using the square loop equation is conservative, over estimating the small-loop voltage. The equations in the Perkins report will produce more accurate results [1.4].

Work stands are often constructed of large metal beams for strength. The beams force the magnetic fields to go around the metal. To account for all the flux, the area of the large loop formed by the beams should be estimated by using the centerline of the beam. (See Figure 2.9.) In most cases, the induced voltage from the shorted loop usually dominates the open-loop voltage. This effect of the beam width is included in the simplified equation through the wire radius term, R.

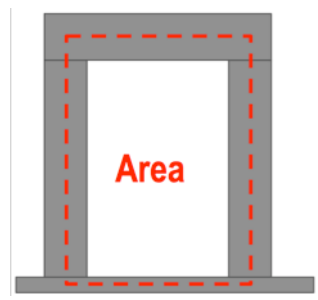


Figure 2.9. Effective area of shorted loop.

In the next section, the laboratory study to validate the simplified equation will be presented.

3 Experimental Validation

Because the equations might be used for critical calculations, a laboratory study was completed to validate the accuracy and limitations of the simplified equations and the full analytical treatment / modeling in reference 1.4. The lightning current changes relatively slowly, around a microsecond, and the lightning wavelength is much greater than the dimensions of the loops. Therefore, we were able to use a 2-meter high transverse electromagnetic (TEM) cell to created known reasonably spatially-uniform magnetic fields to excite our loops. The study was performed in the frequency domain from 10 kHz to 1 MHz where we could accurately characterize relatively small loops. Depending on the frequency, they generate very low voltages, less than a millivolt even with the drive amplifier.

The measurement setup is shown in Figure 3.1. Because the sensitivity of the loop drops with frequency, a broadband amplifier is needed to increase the current, magnetic field, produced by a sine waveform generator. The output of the TEM cell was terminated in a short to increase the current level. A calibrated B-dot sensor measured the magnetic field. A digital scope using the averaging function measured the loop voltages. The current into the cell was also measured to track the gain of the amplifier.

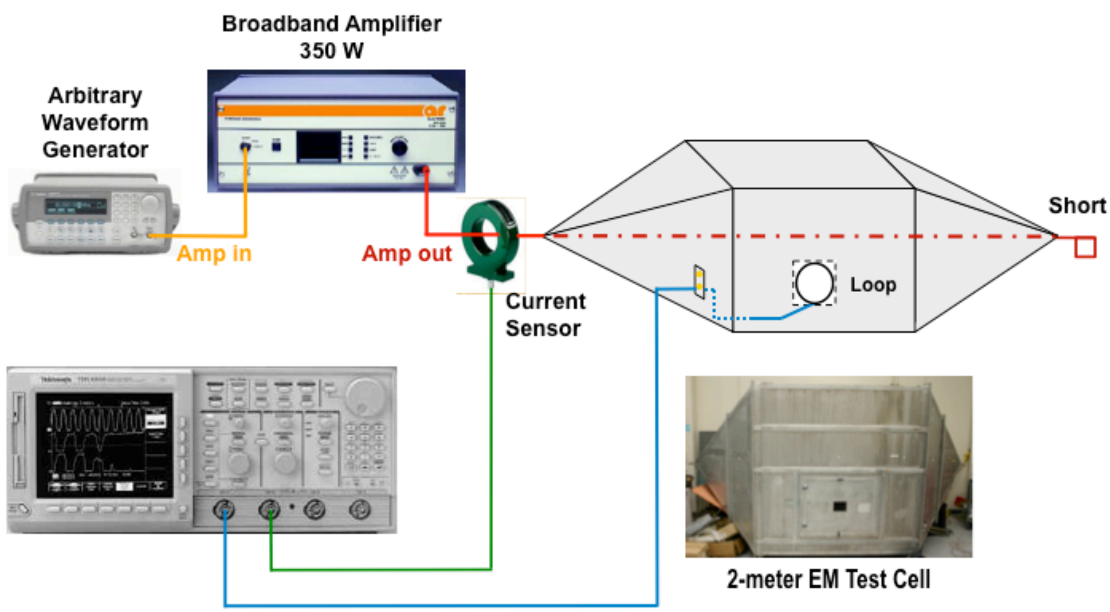


Figure 3.1. Test setup for measuring magnetic field coupling to loops.

A number of loop configurations were checked including individual loops, open loops outside and inside of the shorted loop, and gap between the large and small loops. Intermediate measurements, such as the current in the shorted 1 ft^2 square loop, were completed.

The current in the shorted loop is shown in Figure 3.2. A Tektronix CT2 sensor with a bandwidth of 1.2 kHz to 200 MHz measured the current. The full analysis was used in the current calculation. The small variations in the loop current for both lines were caused by changes in the magnetic field. Since the current in the TEM was being measured, there was little reason to precisely control the magnetic field. There is good agreement between the measured and theoretical current above 200 kHz. Below 200 kHz, the sensor insertion impedance likely caused the divergence between the theoretical and measured currents.

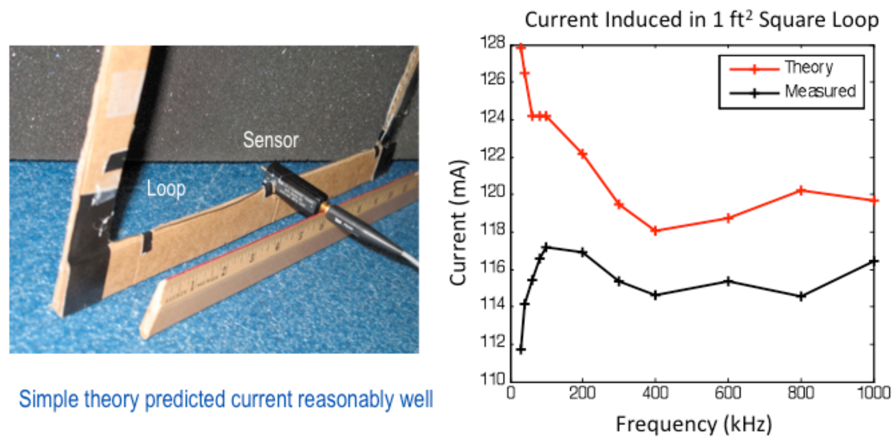


Figure 3.2. Measured current in shorted loop agrees reasonably well with theory.

The test configurations for an open-small loop (2.7 in^2) outside of the shorted-large loop are shown in the photographs in Figure 3.3. The touching loops produced a larger voltage than the configuration with a 2 mm gap. The rate-of-change of the magnetic field density is about 11 Wb/m^{2}s. The predicted and measured voltages shown in the plot match nicely. The calculated voltages from the simplified equation and full analysis, and the experimental data matched within 5% for the touching case at 1 MHz. The data at the lower frequencies, less than 200 kHz, has more measurement noise; nonetheless the agreement was good.

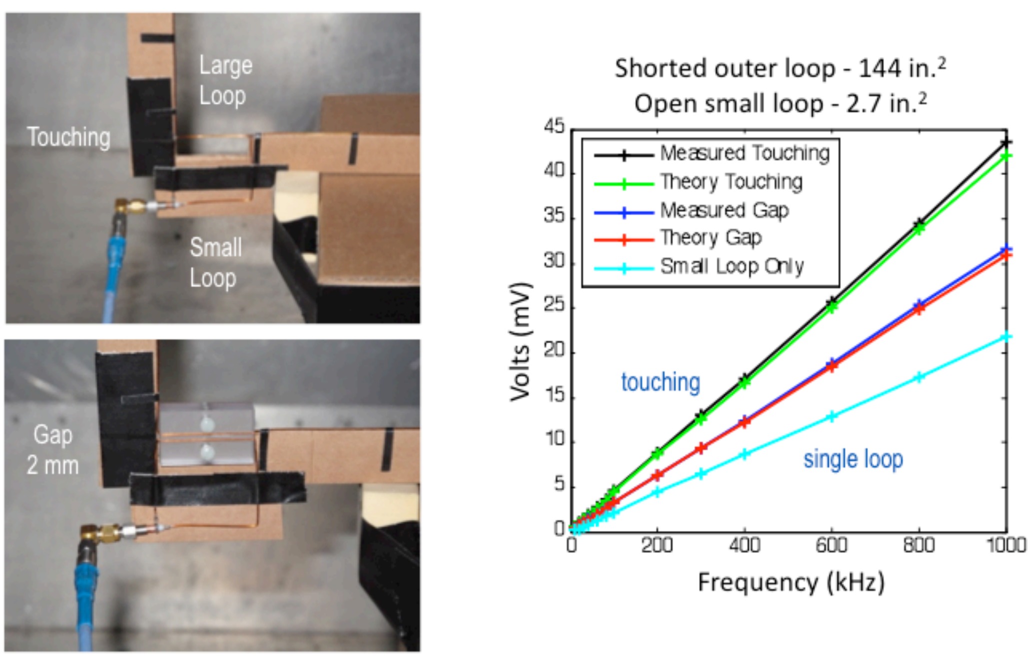


Figure 3.3. An open-small loop near a shorted-large loop will produce more voltage than an isolated small loop.

When an open loop is placed inside of the shorted loop, the two terms in eq. 2.6 have opposite signs. The voltage induced in the open loop by the external field is 180° out of phase from the voltage induced by the flux of the shorted loop. The total open-loop voltage could be less than from an isolated loop. However, the corner location is special and has twice the magnetic density generated by the two orthogonal wires. In the last example, the open loop is placed in an inside corner of the shorted-large loop. (See Figure 3.4.) The total induced voltage is higher than from the isolated loop, but is still less than produced by the open loop on the outside. The application of the simplified equation is more difficult when the open loop is inside the shorted loop in the corner.

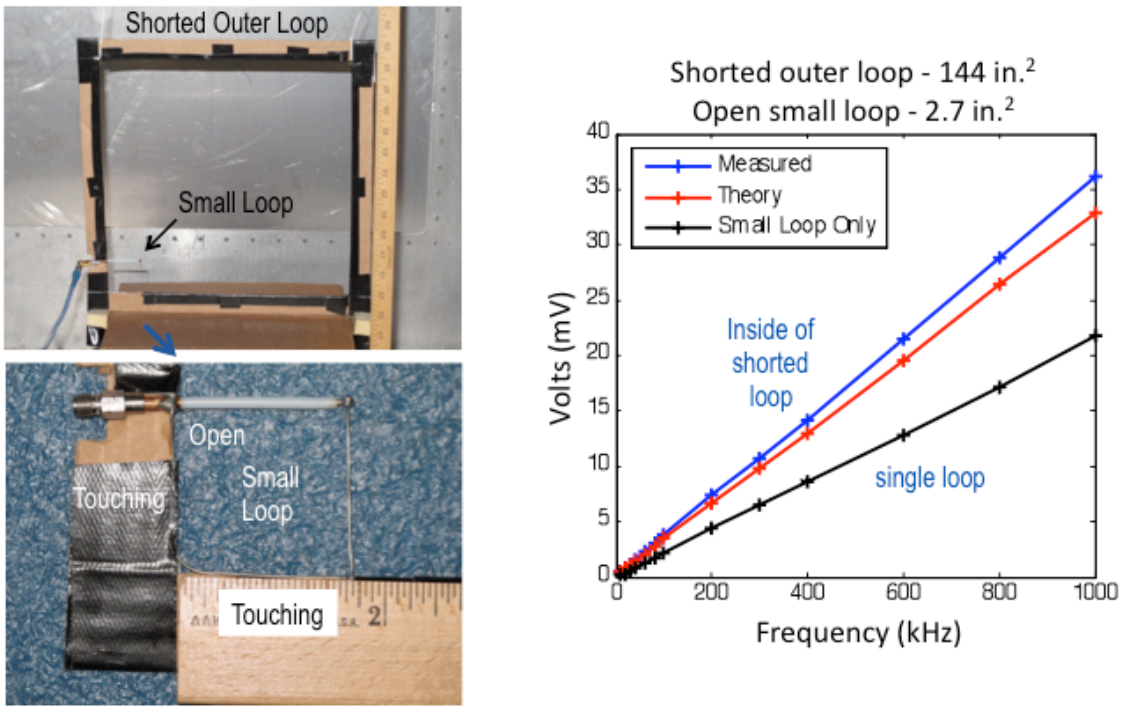


Figure 3.4. The open-small loop situated in an inside corner of the shorted-large loop can produce more voltage than an isolated open-small loop

4 Conclusion and Summary

A detonator cable formed into a loop will produce common-mode voltages when exposed to changing magnetic fields (dB/dt). If the cable is near or is a part of a shorted-large loop, the cable voltage could be significantly higher. A full analysis of the interaction between the loops and the induced voltage is very complex. A simple equation was derived from analysis of the magnetic fields around the shorted loop. The equation assumes the loops are square, and the only parameters required are dB/dt, size length of the loops, and the cable radius.

The simplified equation can be adapted to rectangular or irregularly shaped loops to turn out reasonable voltage estimates. They can serve as a screening level to determine if detailed analysis is required. If the safety margin is large, no further analysis is needed.

The full analysis may also be required if the work stand geometry is very complicated, e.g., three or more loops, there are objects in the loops, or there is the possibility of insulator breakdown. (See Figure 4.1) The full analysis can determine the effect of magnetic field polarization and arc energy.

The insight provided by the simplified equation could help guide the design of work stands that reduce rather than increase the threat posed by a lightning strike.

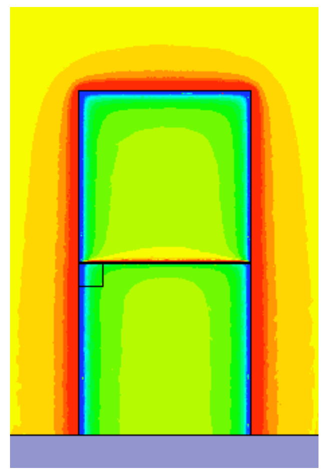


Figure 4.1. Multiple loop interactions may require full analysis.

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